ria percurrit lineolam IK, corpus aliud in circulo in eadem distantia motum percurret arcum $\frac{b}{a} \times KN$; & Area sectoris Circuli & Trajectoriæ simul descriptæ erunt $\frac{b}{a} \times KN \times \frac{1}{2} CN$ & $KN \times \frac{1}{2} CN$, quæ duæ Areæ sunt in ratione data, scil. ut b ad a. Adeoque ubi est $a = b$, uti fit in Spirali Hyperbolica, Area sic descripta erit semper æqualis Areæ sectoris circularis in æquali tempore descriptæ.

November 24. 1713

III. *Rules for correcting the usual Methods of computing Amounts and present Values, by Compound as well as Simple Interest; and of stating Interest Accounts. Offer'd to Consideration, by* Thomas Watkins, *Gent. F. R. S.*

I. Of Compound Interest.

THE Supposition whereon the Method of computing by Compound Interest is founded; *viz.* That all Interest Money, Rents, *&c.* are or may be constantly receiv'd, and put out again at Interest, the Moment they become due, without any Charge, or Trouble, being impracticable; therefore all Computations by this Method (except of Fee-Simples or other Perpetuities) must needs be erroneous. Thus for Instance, the Amount of a Sum of Money, or Annuity, for want of Deductions out of the Profits, for the unavoidable Trouble, Charge, and Delay in the Management, will be too great: and for the same reason, the present value of a Sum of Money payable in any time to come, will be too little; also the present value of an Annuity (being only the Amount of the difference between the Annuity, and Interest of the said present value) will be too much. But in long terms of Years, as that difference becomes less so does the Error; as the term is great-

greater; and in Fee Simples it vanishes; the contrary to which happens in Amounts of Sums of Money, and Annuities.

All which is propos'd to be rectify'd, only by a just reduction of the Rate, and Annuity; (which is done by deducting so much *per Cent.* thereout, as the whole Trouble, and Charge of Management is suppos'd to amount to, and reducing the Remainder, by a Discount equivalent to the suppos'd loss of time) and then by working with the Rate so reduc'd, for Sums of Money, and with the Rate and Annuity reduc'd in the like proportion for Annuitys, according to the common Method of Compound Interest; as follows. Put r for the rate of Interest of 1 *l.* c for the Charge and Trouble of the Management of 1 *l.* Then is $r - cr =$ the Rate after deducting the said Charge, $=$ (putting d for $1 - c$) dr. And for the Discount put t for the time lost, that is for such part of the Period of time in which the Payments are made (whether Yearly, ½ Yearly, Quarterly, or otherwise) as is suppos'd to be spent in receiving and putting them out again at Interest. Then, dtr, being $=$ the Interest of 1 *l.* for that time; say, as $1 + dtr : 1 :: dr : \frac{dr}{1 \times dtr} =$ (putting e for $1 + dtr$) $\frac{dr}{e}$, which is equal to the reduc'd Rate, near enough for practice, for which put $\mathfrak{r}$. But if the utmost accuracy be requir'd, the Discount itself must be made with regard to the like loss of time, which is done by a Series of Discounts rais'd thus; $e(=1 + tdr) : tdr :: dr : \frac{td^2 r^2}{e} :: \frac{t d^2 r^2}{e} : \frac{t^2 d^3 r^3}{e^2}$ &c.

Whence $dr - \frac{td^2 r^2}{e} + \frac{t^2 d^3 r^3}{e^2} - \frac{t^3 d^4 r^4}{e^3}$ &c. $=$ (putting q for $\frac{tdr}{e}$) $\overline{1 - q + q^2 - q^3 \text{ \&c.}} \times dr. = \frac{dr}{e} (= \overline{1 - q} \times dr) + \overline{q^2 - q^3 \text{ \&c.}} \times dr$, is $= \mathfrak{r}$, $=$ the true Rate reduced. Put $s = 1 + \mathfrak{r}$, $n =$ the time, $p =$ the present Sum or Value, $m =$ the Amount, Then

Then will $\overline{1+\frac{dr}{e}+q^2-q^3\ \&c. \times dr}\big|^n \times p = \overline{1+\mathfrak{r}}\big|^n \times p = ps^n$, be exactly $= m$: But $\overline{1+\frac{dr}{e}}\big|^n \times p = m$ is sufficient for practice.

And for the Amounts and present values of Annuitys.

Put $A =$ Annuity *per annum.*
$a =$ Annuity $\frac{1}{2}$ yearly, quarterly, *&c.*
$R =$ Yearly rate of Interest of 1 *l.*
$r =$ Rate $\frac{1}{2}$ yearly, quarterly, *&c.*
$\mathfrak{r} =$ Reduc'd rate yearly, $\frac{1}{2}$ yearly, quarterly, *&c.*
$n =$ Number of Years, $\frac{1}{2}$ years, quarters, *&c.*

Then will $A\frac{\mathfrak{r}}{R} = a\frac{\mathfrak{r}}{r}$ be $=$ reduc'd Annuity taken yearly $\frac{1}{2}$ yearly, quarterly, or otherwise; and by Compound Interest 'twill be $\frac{\overline{1+\mathfrak{r}}|^n - 1}{\mathfrak{r}} \times a\frac{\mathfrak{r}}{r} = \frac{\overline{1+\mathfrak{r}}|^n - 1}{r}\ a = \frac{\overline{1+\mathfrak{r}}|^n - 1}{R}\ A = \frac{s^n - 1}{R} A = m$, and $\frac{s^n - 1}{R\, s^n} A \left(= \frac{m}{s^n}\right) = p$. Whence the Theorems for solving all the other Cases are easily deduc'd. And if the Rate be requir'd, when 'tis for a Sum of Money, the Solution is obvious: when for the Amount or Value of an Annuity, since $\overline{1+r}|^n = \frac{mr+a}{a} = \frac{a}{a-pr}$ are the Equations whence Theorems for the Rate are usually deriv'd, which by this Correction become $\overline{1+\mathfrak{r}}|^n = \frac{mr+a}{a} = \frac{a}{a-pr}$. That the same $\mathfrak{r}$ may be had on both sides the Equation, put $u\mathfrak{r}$ for r, and 'twill be $\overline{1+\mathfrak{r}}|^n = \frac{mu\mathfrak{r}+a}{a} = \frac{a}{a-pu\mathfrak{r}}$: then, by the Rate assum'd as near the truth as may be, find the value of $u \left(= \frac{1}{d} + tr = \frac{1}{d-td\mathfrak{r}}\right)$ and in any Theorem for the

Rate,

Rate, putting *m u* for *m*, and *p u* for *p*, the result will be the Rate reduced nearly: and by repeated Operations correcting r and thereby *u*, the true r, and thence *R* the whole Rate, may be found.

The only difficulty that remains, is the right assuming the Quantities *c* and *t*, the impossibility of doing which with perfect Exactness, I suppose to be the reason why neither this, nor any Method of Correction to the like purpose, has yet been taken notice of by the Writers on this Subject; and what may therefore be very likely to be objected to this. But the same Objection I take to be of equal force against the Estimates of any other Uncertainties whatsoever, as Estates for Lives, Insurances *&c.*

First then, for the Quantity *c*, which is put for the Trouble and Charge of Management, *viz.* of collecting and placing out the Money on good Security, together with all Contingencies attending the same, as travelling Charges, Expences, Attorney's Bills, *&c.* of all which, the principal Article is the Charge of Collection or Receiver's Fees, which is commonly a fix'd Rate, customarily allow'd in the Place, or upon the Estate it self out of which the Purchase is made, if it be a Rent; and for Interest Money, or any other Annuity, the like Estimate is to be made, whether the Proprietor acts for himself, or by another. Then for the Charge of placing out the Mony at Interest when receiv'd; though this be for the most part defray'd by the Borrower, yet because it highly concerns the Lender to see it be securely done, there are usual Allowances made to Agents and Scriveners, to encourage their Care and Fidelity therein; besides the Time, Expence and Trouble of the Proprietor himself, in finishing Contracts, inspecting Securities *&c.* and whatever is sav'd in this Article, we must suppose to be fully made up by an Equivalent Degree of Risque in the Security.

In the next place, for the loss of time; though 'tis also impossible for this to be exactly ascertain'd, nor perhaps so nearly as the former, since it depends very much on the Diligence of the Manager: Yet if the usual times of Payment of of the particular Rent or Annuity to be purchaſed, with a moderate degree of diligence in the Manager, and the uſual indulgence practis'd by Men of Buſineſs in this Caſe towards one another be obſerv'd, a reaſonable Eſtimate may be alſo made of the loſs of time; In which 'tis to be noted, that Intereſt Money being uſually paid in ſmall Sums, when any Sum of Money to be made up of ſeveral ſuch Payments, is intended to be put out at Intereſt, the whole muſt lie dead till the laſt Payment be made; alſo that the Principal lies dead ſometimes as well as the Intereſt; and that on the other hand, to ſave time, Borrowers may be found out, and treated with during the time of Collection; but this Advantage is in a great Meaſure loſt by the difficulty of fixing the time or *Quantum* of a Loan, till the whole be paid in. Note alſo, that if the Charge of Collection, or Loſs of time, on the Rent, or Annuity, of any particular Eſtate or Place, be found to differ from that of the Intereſt of the Purchaſe Mony, and ſo much exactneſs be required, as that the Computation be made with regard to ſuch difference: It muſt be done, either by aſſuming a proper Medium for both, or more accurately thus: For the reduc'd Rate of the $\left\{\begin{matrix}\text{Annuity}\\ \text{Intereſt-Mony}\end{matrix}\right\}$ put $\left\{\begin{matrix}\mathfrak{z}\\ \mathfrak{r}\end{matrix}\right\}$ Then $r : \mathfrak{z} :: a : a\frac{\mathfrak{z}}{r}$,

and $\mathfrak{r} : \overline{1+\mathfrak{r}}|^{n} - 1 :: a\frac{\mathfrak{z}}{r} : \frac{\overline{1+\mathfrak{r}}|^{n} - 1}{r} a \times \frac{\mathfrak{z}}{\mathfrak{r}} = m = \overline{1+\mathfrak{r}}|^{n} \times p$.

To give a Specimen of this Method in Numbers, firſt the Quantities *c* and *t* are to be aſſum'd, which are not here to be accomodated to any particular Place or Eſtate, but to be taken in general: And firſt for *c* the Charge of Management; the uſual Rates of Collectors Fees in theſe Kingdoms, as I am inform'd, are 6*d.* 12*d.* and 18*d. per* Pound, but the moſt uſual 12*d.* which is 5 *per Cent.* However to be within compaſs, I ſhall take 4 *per Cent.* for the Medium, including what further trouble and charge may attend the Receipt of the Mony, besides Receivers Fees; and 2 *per Cent.* for all the other Charges before mention'd, in placing it out at Interest, both which make 6 *per Cent.* so that c is $=0, 06$, and d $(=1-c)=0, 94$. Next for t the Loss of Time; since few Annuities are paid yearly, and a Discount being given for the Loss of Time, we are to lose no more than is discounted for; therefore I choose Half-yearly Payments for the Examples, being the most usual, which with little Alteration may serve for quarterly; and considering the before-mention'd Circumstances relating to the Time, I look upon two Months the least, and seven or eight Months the most, that can well be suppos'd to be spent, one Time with another, in receiving and putting out the Mony, upon a moderate Management; between which the Medium is about four Months and half, which being $\frac{3}{4}$ of $\frac{1}{2}$ a Year, gives $\frac{3}{4}=t$: and if $\frac{dr}{e}$ be $=\mathfrak{r}$, the yearly Rates of 4, 5, 6, 8 and 10 *per Cent.* will produce for Half-yearly Rates reduc'd of 1 *l.* 0.018539, 0.023093, 0.027616, 0.036569 and 0.0454 each $=\mathfrak{r}$. But if $\mathfrak{r}$ be $=\frac{dr}{e}+\overline{q^2-q^3}$ &c. $\times dr$, 'twill be, 0.01854219, 0.0230999155, 0.027627775, 0.036596289 and 0.04545235, each $=\mathfrak{r}$, (so that each Rate loses by this Estimate about $\frac{1}{12}$ Part.) Whence the following Amounts, and present values of 1 *l.* *per Annum* computed Half-yearly, are produc'd, and compared with those of the usual Method computed yearly, to agree with the common Tables.

Years.	Amounts of 1 l. per An. at 5 per Cent. by Comp. Int.	Co. Int. cor.	Differences.	Amounts of 1 l. per An. at 6 per Cent. by Comp. Int.	Co. Int. cor.	Differences.
5	5,52563	5,13104	,39459	5,63709	5 22138	,41571
10	12,57789	11,57846	,99943	13,18079	12,07854	1,10225
20	33,06595	29,85996	3,20599	36,78559	32,91054	3,87505
30	66,43885	58,72546	7,71339	79,05819	68,83976	10,21843
40	120,79977	104,30079	16,49898	154,76197	130,80733	23,95464
60	353,58372	289,88164	63,70208	533,12818	422,01429	111,11389
80	971,22882	752,53436	218,69446	1746, 5999	1288, 2484	458,3515
100	2610,02516	1905,92671	704,09845	5638, 3680	3864, 9753	1773,3927
	The fame at 8 *per Cent.*			The fame at 10 *per Cent.*		
5	5,86660	5,40633	,46027	6,10510	5,59705	,50805
10	14,48656	13,15092	1,33564	15,93743	14,32680	1,61063
20	45,76196	40,13759	5,62437	57,27500	49,17930	8,09570
30	113,28321	95,51623	17,76698	164,49402	133,96428	30,52974
40	259,05652	209,15737	49,89915	442,59257	340,21900	102,37357
60	1253,2133	920,90040	332, 3129	3034,81648	2062,57167	972,24481
80	5886,9354	3918,0578	1968, 8776	20474,0027	12255,3348	8218, 6679
100	27484,5157	16539,0989	10945.4168	137796,127	72575,3926	65220,7344

Years.	Pref. Values of 1 l. per An. at 5 per Cent. by Comp. Int.	Co. Int. cor.	Differences	Pref. Values of 1 l. per An. at 6 per Cent. by Comp. Int.	Co. Int. cor.	Differences
5	4,32948	4,08343	,24605	4,21236	3,97582	,23654
10	7,72174	7,33314	,38860	7,36009	7,00322	,35687
15	10,37966	9,91935	,46031	9,71225	9,30843	,40382
20	12,46221	11,97753	,48468	11,46992	11,06373	,40619
30	15,37245	14,91902	,45343	13,76483	13,41805	,34678
40	17,15909	16,78200	,37709	15,04630	14,78309	,26321
50	18,25593	17,96190	,29403	15.76186	15,57456	,18730
70	19,34268	19,18247	,16021	16,38454	16,29953	,08501
100	19,84791	19,79231	,05560	16,61755	16,59510	,02245
F.S.	20,00000	20,00000	,00000	16,66666	16,66666	,00000

Note, That this Correction is alfo applicable to the Valuations of Eftates for Lives, in which the firft Step being to find an Equivalent in Years of Continuance, brings them to the Cafe of Eftates for Years.

 Of

II. Of Simple-Intereſt.

The abſurdity of the Suppoſition, on which the uſual Method of computing preſent Values by Simple-Intereſt, is founded, *viz.* That the Rent or Annuity is conſtantly received, and put out again at Intereſt, as it becomes due; but that the Intereſt of the Purchaſe Mony lies dead during the whole Term, is ſo apparent, and the Errors ariſing from it ſo groſs, that the Writers who have laid down this Method, have at the ſame time caution'd againſt the Uſe of it for any more than 6 or 7 Years, the Error for that time being not conſiderable.

The ſame Suppoſition does alſo occaſion the miſcomputation of Amounts, or rather the miſapplication of them to their proper Caſes. Wherefore, ſince the Simple-Intereſt of Mony is of equal Value *pro rata*, and of the ſame regard with Rents or Annuitys, being each the Original Profits iſſuing alike from a principal Stock, Eſtate or Value, and equally improveable; This general Rule may ſerve for a juſt Correction of this Method, *viz.* That ſuppoſing in any Caſe an Intereſt ought to be, or not to be allowed to either of thoſe Profits, the ſame be done in the like Caſe to the other. Thus, in the Caſe of Debts, or Amounts of Sums of Mony, Rents, or Annuitys for the time paſt, its uſual in practiſe to allow no Intereſt to either: For though the Law, to curb the exorbitant Avarice of Uſurers, and for other Reaſons, does more expreſly diſallow Intereſt upon Intereſt for a Debt; our Courts both of Law and Equity, as I am inform'd, will be as far from allowing the Charge of Intereſt againſt a Tenant for Rent in Arrear; except on a *nomine pœnæ* (which is now become almoſt obſolete) ſo that in this Caſe (putting a and r for the Annuity and Rate yearly, Half-yearly, or otherwiſe) as $prn + p$ is the Amount of a Sum of Mony, ſo is an the Amount of an Annuity or Rent in Arrear, and not $\overline{\frac{n-1}{2} r + 1} \times an$ as Arithmeticians commonly make it. But in the Computation of preſent Values, or Amounts for the time to come,

come, the ſame being made on the Expectation of a conſtant regular Income of the Profits, without any extraordinary Interruption, an Intereſt ought to be allow'd to both, eſpecially in preſent Values, which are found by ſetting the Amounts of both againſt each other: ſo that in theſe Caſes, putting $x = n - 1$, if $\overline{\frac{1}{2}xr + 1} \times an$, be made the Amount of a Rent or Annuity; then $\overline{\frac{1}{2}xr + 1} \times prn + p$, will be the proper Amount of a Sum of Money, and not $prn + p$: and conſequently $\frac{\overline{\frac{1}{2}xr + 1} \times an}{\overline{\frac{1}{2}xr + 1} \times rn + 1} = p$ will be the preſent Value of a Rent or Annuity, the ſubſequent Intereſt being remitted on both ſides in lieu of the Loſs of Time and Charge of Management; which ſuch as are apt to depreciate long Futurities, may think the propereſt Method of approaching the true Value; but I rather look upon the former Method of Compound Intereſt corrected as more exact, as well as more general, the Intereſt remitted in this, being in ſhort Terms leſs, and in long Terms more than an Equivalent for the Trouble, Charge and Delay in the Management. But it is however the moſt exact of any of the Methods, that have yet been deduc'd from Simple-Intereſt. The reduc'd Rate may alſo in ſome Caſes be properly made uſe of for Amounts, but not for preſent Values, except for ſhort Terms; and then, ſince $r : a :: \overline{\frac{1}{2}x\mathfrak{r} + 1} \times \mathfrak{r}n : \overline{\frac{1}{2}x\mathfrak{r} + 1} \times an\frac{\mathfrak{r}}{r}$, 'twill be $\overline{\frac{1}{2}x\mathfrak{r} + 1} \times an\frac{\mathfrak{r}}{r} = m = \overline{\frac{1}{2}x\mathfrak{r} + 1} \times p\mathfrak{r}n + p$, and $\frac{\overline{\frac{1}{2}x\mathfrak{r} + 1} \times an}{\overline{\frac{1}{2}x\mathfrak{r} + 1} \times \mathfrak{r}n + 1} \times \frac{\mathfrak{r}}{r} = p$.

Examples of this Method compar'd with thoſe of the former, will ſtand as follows; in which all is computed Half-yearly, except the laſt Column of Compound Intereſt.

Amounts of 1 *l.* at 5 *per Cent.* computed 6 ſeveral Ways.

Years.	1	2	3	4	5	6
	Simple Int.	*Id. for Bonds*	*Sim. Int. cor.*	*Id. by the red. rate*	*Co. Int. cor.*	*Comp Int.*
	$prn + p = m$	Id. till $prn = p$	$\overline{\frac{1}{2}\times r+1}\times prn + p = m$	$\overline{\frac{1}{2}\times r+1}\times pr\,r + p = m$	$\overline{1+r}^{n}\times p = m$	$\overline{1+R}^{N}\times p = m$
5	1,25	1,25	1,27813	1,25501	1,25655	1,27628
10	1,5	1,5	1,61875	1,56338	1,57892	1,62889
20	2,	2,	2,48750	2,34021	2,49300	2,65330
30	2,5	2,	3,60625	3,33048	3,93625	4,32194
40	3,	2,	4,97500	4,53419	6,21504	7,04000
60	4,	2,	8,46250	7,58194	15,49408	18,67919
80	5,	2,	12,95000	11,48345	38,62672	49,56144
100	6,	2,	18,43750	16,23874	96,29634	131,50126
	Amounts of 1 l. *at* 6 per Cent. *by the ſame Theorems.*					
5	1,3	1,3	1,3405	1,31063	1,31328	1,33822
10	1,6	1,6	1,7710	1,69758	1,72471	1,79084
20	2,2	2,	2,9020	2,70048	2,97463	3,20713
30	2,8	2,	4,3930	4,00870	5,13039	5,74349
40	3,4	2,	6,2440	5,62223	8,84844	10,28572
60	4,6	2,	11,0260	9,76525	26,32086	32,98769
80	5,8	2,	17,2480	15,12954	78,29488	105,79599
100	7,	2,	24,9100	21,71510	232,89852	339,30208
	Amounts of 1 l. *at* 10 per Cent. *by the ſame Theorems.*					
5	1,5	1,5	1,6125	1,54749	1,55970	1,61051
10	2,	2,	2,4750	2,30157	2,43268	2,59374
20	3,	2,	4,9500	4,42951	5,91793	6,72750
30	4,	2,	8,4250	7,38381	14,39643	17,44940
40	5,	2,	12,9000	11,16449	35,02190	45,25925
60	7,	2,	24,8500	21,20493	207,25717	304,48165
80	9,	2,	40,8000	34,55084	1225,5335	2048,4003
100	11,	2,	60,7500	51,20222	7258,5398	13780,6127

Years

Years.	*Pres. Values* of 1 l. per An. *at* 5 per Cent. *by*			*Pres. Values* of 1 l. per An. *at* 6 per Cent. *by*		
	Sim. In. cor.	*Co. Int. cor.*	*Comp. Int.*	*Sim. In. cor.*	*Co. Int. cor.*	*Comp. Int.*
5	4,35208	4,08343	4,32948	4,23349	3,97582	4,21236
10	7,64478	7,33314	7,72174	7,25579	7,00322	7,36009
15	10,10821	9,91935	10,37966	9,39341	9,30843	9,71225
20	11,95980	11,97753	12,46221	10,92350	11,06373	11,46992
30	14,45407	14,91902	15,37245	12,87275	13,41805	13,76483
40	15,97990	16,78200	17,15909	13,99744	14,78309	15,04630
50	16,96682	17,96190	18,25593	14,69544	15,57456	15,76186
70	18,10986	19,18247	19,34268	15,47252	16,29953	16,38454
100	18,91525	19,79231	19,84791	15,99759	16,59510	16,61755
F.S.	20,00000	20,00000	20,00000	16,66666	16,66666	16,66666

The Theorems to the preceding Columns of Amounts (of which the fourth aud fifth are infinitely variable in the Result, by assuming *c* and *t* in the reduc'd Rates at Pleasure) may serve to answer all the simple Cases of Amounts that occur in Business: To instance in some,

1. The first Column contains the Amounts of such Debts, or Sums of Mony as carry a simple Interest till the Principal be paid.

2. The second Column answers the common Case of Debts due by Bond, that by Law are allow'd not to exceed the Penalty, which is generally double the principal Debt.

3. The third Column answers the Case of a Security or joynt Obligor, that has duly and constantly paid the Interest, and at last the principal Sum of a Debt, from which he has a Counter-Bond from the principal Debtor to save him harmless, against whom he may make his Charge from this Column.

4. In case the Parties shall agree that the Debt shall lie for any time certain, or uncertain; and for the much greater ease, advantage and satisfaction of both of them, no Interest to be call'd for, till the Principal it self is paid; but to carry Interest as it becomes due, the Lender allowing for the time and charge he must have been at, in receiving and putting out his Interest, the fourth Column will fit this Case, or else the fifth

as

as a greater or leſs degree of Lenity is agreed upon in favour of the Borrower.

5. The fifth Column is alſo proper in the following Caſe, *viz.* if it be demanded, what Eſtate in reverſion, after a certain number of Years, any Sum in Hand will Purchaſe; the firſt ſtep being to find the Amount of that Sum to the time the Reverſion commences, its had in this Column.

6. The laſt Column gives the Amount of a Sum of Money, according to the common Method of Compound Intereſt, but being computed with that extraordinary rigor as has been ſaid, (except ſome ſmall allowance for the loſs of time, by being done Yearly) 'tis hardly ſuitable to any Caſe.

Other Caſes might be enumerated, to which the foregoing Theorems might be equitably apply'd; beſides ſuch extraordinary ones, wherein it may appear to Arbitrators, or a Court of Equity, that either Partie deſerves Favour, either by way of Compenſation for Injuries ſuffer'd from the other, by means of any fraudulent or oppreſſive Practices, or otherwiſe, for which no other redreſs is provided.

III. Of Intereſt Accounts.

The Inequality of the uſual Method of ſtating Intereſt Accounts, as practis'd in our Courts of Equity, will beſt appear by an Example, for which I ſhall take the common general Caſe of an Intereſt Account to be ſtated on a Mortgage, *viz.* ſuppoſe 1000 *l.* to be let out at 6 *per Cent.* on a Mortgage of 120 *l. per annum*, payable Half Yearly, and the Mortgagee after five Years end, to have Poſſeſſion till the Arrear of Intereſt, accruing Intereſt and Principal be diſcharged: *Quære*, How long that will be? ſuppoſing alſo, for the ſake of brevity in the Account, the Payments to be equally, and punctually made as they become due. By the Chancery Method, the Rent is firſt apply'd to diſcharge the Arrear of Intereſt; and then the remainder of every Half-Year's Rent, after deducting the ſame Half-Years Intereſt, is apply'd towards the Diſcharge of of the Principal, and thereby the Principal and Intereſt continually leſſens, till the whole be paid off. Now by this means the Mortgagee, after the Arrear is diſcharged, pays Comp. Intereſt, with the utmoſt rigour, for ſo much *per annum* of the Rent, as exceeds the Intereſt of the whole Principal Mony, and receives but Simple Intereſt for his Debt; which, however ſtrange it may ſeem, is eaſily prov'd, by applying the proper Theorems of Simple and Compound Intereſt to this Caſe, in which the Annuity, Principal Money, Rate and Arrear of Intereſt are given, and the time requir'd; the reſult being the ſame with that of the Chancery Method, except a very ſmall difference only when any part of the time is expreſs'd by a Fraction: *viz.* putting L for Logarithm, $\alpha = a - pr = 30$, $s = 1 + r = 1{,}03$, $t =$ time of contracting the Arrear $= 10$ Half-years, $n =$ any number of $\frac{1}{2}$ Years ſpent in diſcharging the whole or any part, $N =$ number of Years required; the Equation for the Arrear will be $prt + prn = an$; and for the Principal and accrueing Intereſt $prn + p = prn + \frac{s^n - 1}{r} \times \alpha$. Whence

$$\frac{prt}{2\alpha} + \frac{La - L\alpha}{2Ls} = N = 16{,}7249 \text{ Years} = \text{the time demanded};$$

i. e. $\frac{prt}{2\alpha} = 5$ Years $=$ the time of diſcharging the Arrear, and

$\frac{La - L\alpha}{2Ls} = 11{,}7249$ years, $=$ the time in which the Principal and accrueing Intereſt is diſcharged; during which its evident, the Mortgagee pays full Compound Intereſt for 60 *l.* *per annum* of the Rent. For the Correction of which inequality, in the firſt Place, to the end that neither Branch may exceed, or be depriv'd of its due Profits; This general Rule is propos'd as neceſſary to be always obſerv'd, *viz.* That Amounts of the Produce on each ſide be ſtated ſeparately, and ſet againſt each other in the Account, in order to a Ballance: And in the common Caſes of Mortgages, Government, and

Stock Securities, *&c.* where the Debt is paid off by a Rent, Annuity, Pension, Dividend, or other Payments issueing in the same manner, and with the like trouble, charge, *&c.* as Interest Money does; I presume this Rule will also be easily admitted, *viz.* That the same equitable Advantage be impartially allow'd on both sides; for which the Method of Simple Interest, as corrected under the foregoing Head, seems truly adapted; whereby the Original Profits on each side are suppos'd to be deem'd, either as Interest, or else as Principal Mony; and since the Amounts both of an Annuity and Sum of Mony, for the time past, as there stated, on the first of these Suppositions (t being there $= 0$) are likewise vouch'd by our Laws, and the practice of our Courts, to be good when separately us'd; I think its very evident, that the Account ought to be stated by setting those Amounts against each other thus, $prt + prn + p = an$, (whence $\frac{1 + tr}{2\alpha} p = N, = 21{,}6666$ Years) and that this Method is most proper for general Use, in the Cases mention'd: Unless it should be thought fit, in consideration of the various Ways found out for the ready improvement of Mony, to allow a further Advantage on both sides, by charging the Original Profits as Principal Mony, and giving a Simple Interest there to, which still falls short of the Advantage allow'd to Rents by the Chancery Method. And this is to be done two ways, *viz.* either by applying an Amount of Rent to pay off the Arrear first, and afterwards another Amount of Rent to discharge the Principal, and accrueing Interest; or else by letting the Profits with all Arrears and other Charges run on at Simple Interest on each side, till the end of the Term: *viz.* putting $x = n - 1$, $y = t - 1$, $\alpha = a - pr$, $f = \overline{2 - r} \times \alpha$, $g = f - 2ptr^2$, $\mu = \overline{\frac{1}{2}yr + 1} \times prt =$ Arrear of Interest; by the first of these 'twill be, for the Arrear, $\mu + ptr^2 n = \overline{\frac{1}{2}xr + 1} \times \alpha n$, and for the Principal and accruing Interest, $\overline{\frac{1}{2}xr + 1} \times \alpha n = p$; Whence

$\surd 8 \mu \alpha$

$\frac{\sqrt{8\mu a r + g^2} - \sqrt{8 p a r + f^2} - f + g}{4 a r} = N = 18{,}7653$ Years. By the other 'twill be, for the whole, $\mu + ptr^2 n + p = \overline{\tfrac{1}{2} x r + 1} \times a n$, Whence $\frac{\sqrt{\mu + p \times 8 a r + g^2} - g}{4 a r} = N =$ 18,1648 Years. The Lender will have to alledge for the firſt of theſe two Ways, that as the Rent is not hindred by any other parallel Charge from making the utmoſt produce it can, ſo for that reaſon ought his Principal Mony to have the Advantage of the Arrears being firſt diſcharged, which alſo agrees with the Chancery Method in this particular.

Laſtly, another way of ſtating this Account, may be taken from that Notion of Simple-Intereſt, whereby the Annuity only is charged as Principal Mony: and then 'twill be, for the Arrear, $p r t + p r n = a n$, and for the Principal and accruing Intereſt, $\overline{\tfrac{1}{2} x r + 1} \times a n = p$; Whence $\frac{\sqrt{8 p a r + f^2} - g}{4 a r}$ (= $\frac{p r t}{2 a} + \frac{\sqrt{8 p a r + f^2} - f}{4 a r}$) $= N =$ 17,3072 Years; which appears to be ſame with the Chancery Method, only that the Compound Intereſt in that, is turned into Simple in this; and as it ſtill retains part of the ſame inequality, to the Advantage of the Borrower, it ſeems only fit to be obſerved in ſuch Caſes wherein the Borrower may be thought to merit favour, as when the Debt is paid out of the Profits of Trade, ariſing by extraordinary Riſque or Induſtry. But ſince ſuch a Rule of diſtinction is hardly poſſible to be reduc'd to general practice, the uſe of this Theorem ſeems reſtrained to ſuch Caſes only, wherein the Parties themſelves, or a Court of Equity ſhall think it reaſonable.

For a further illuſtration of theſe Rules, the following Specimen is added, which ſhews at ſight, how the Reſults of the ſeveral Methods differ, as the Rent, Arrear, or Rate of Intereſt, is greater or leſs, and conſequently of how much more

or

or leſs concern it is to the Parties, as well as to the due Adminiſtration of Juſtice, to have regard thereto.

The time requir'd in	Years (computed Half-yearly.)			
To diſcharge a Mort. of 1000 *l.* by a Rent of	120 *l. per annum.*		90 *l. per annum.*	
At the Rate of Int. *per annum* of	5 *per C.*	6 *per C.*	5 *per C.*	6 *per C.*
No Arrear of Intereſt.				
By the Chancery Method	10,9141	11,7249	16,4205	18,5835
By the ſame turning the Comp.Int. into Sim. / By Simp.Int.cor. theOrignal Profits charged as Princ. and theArrear contin'd at Int. / firſt diſcharg'd	11,2579	12,3072	17,5335	20,7400
By Simp.Int.cor. theOrignal Profits charged as Intereſt	14,2857	16,6666	25,0000	33,3333
5 Years Arrear of Intereſt.				
By the Chancery Method	14.4855	16,7249	22,6705	28,5835
By the ſame turning the Comp.Int. into Sim.	14,8293	17,3072	23,7835	30,7400
By Simp.Int.cor. theOriginal Profits charged as Princ. and theArrear contin'd at Int.	15,3191	18,1648	24,7148	32,7064
By Simp.Int.cor. theOriginal Profits charged as Princ. and theArrear firſt diſcharg'd	15,5897	18,7653	25 4899	34,8052
By Simp.Int.cor. theOriginal Profits charged as Intereſt	17,8571	21,6666	31,2500	43,3333
10 Years Arrear of Intereſt.				
By the Chancery Method	18,0569	21,7249	28,9205	38,5835
By the ſame turning the Comp. Int. into Sim.	18,4007	22,3072	30,0335	40,7400
By Simp.Int.cor. theOriginal Profits charged as Princ. and theArrear contin'd at Int.	20,3161	25,5998	33,6320	48,0874
By Simp.Int.cor. theOriginal Profits charged as Princ. and theArrear firſt diſcharg'd	21,2895	27,5587	36,1932	53,8107
By Simp.Int.cor. theOriginal Profits charged as Intereſt	21,4286	26,6666	37,5000	53,3333
15 Years Arrear of Int.				
By the Chancery Method	21,6283	26,7249	35,1705	48,5835
By the ſame turning the Comp. Int. into Sim.	21,9721	27,3072	36,2835	50,7400
By Simp.Int.cor. theOriginal Profits charged as Princ. and theArrear contin'd at Int.	26,1353	34,2913	43,9764	65,8386
By Simp.Int.cor. theOriginal Profits charged as Princ. and theArrear firſt diſcharg'd	28,0209	37,7141	48,5159	74,5664
By Simp.Int.cor. theOriginal Profits charged as Intereſt.	25,0000	31,6666	43,7500	63,3333

I muſt alſo obſerve for the ſake of ſuch as are unacquainted with ſpecious Arithmetic, that though for brevitys ſake, the foregoing Theorems and Examples are laid down, and wrought in Algebraic Terms: Yet the ſame Accounts may be ſtated after the Chancery manner it ſelf, according to the ſeveral Principles before deliver'd, and with the ſame Reſults, with this only caution, that (inſtead of a continual deduction of the Rent or Annuity out of the Principal and Intereſt of the Debt, which occaſions the Error before mention'd) the preceding General Rule of ſtating ſeparate Amounts be obſerv'd, which may be done by continually adding the profits together on each ſide, in the ſame manner, as if the Parties were

to

to make a ſeparate Charge againſt each other, which is the rather to be noted, as being the only Courſe that can be taken, in caſe the Sums or Times of payment ſhould differ, but the reſpective Reſults will notwithſtanding be analogous to the above Examples.

All which is ſubmitted to the conſideration of more diſcerning Judgments, eſpecially the Applications of the Rules to particular Caſes, for exemplifying the Theorems. But if any of thoſe Rules or Theorems ſhould be objected againſt, meerly becauſe they tend to introduce ſome Alterations in the preſent practice; I ſhall for anſwer only add, with ſubmiſſion, to what is before ſaid, that in former Ages, when our Laws relating to theſe matters had their riſe, (the Profits of *England* ariſing chiefly from Husbandry and Tillage, and little from Trade,) the Caſh of the Kingdom was but low, the Rates of Intereſt very high to the Advantage of Uſurers, and thoſe ways for the ready Improvement of Money accomodated to all Peoples uſe, not known; (much like to which we are told was the State of the Jewiſh Affairs, when they were forbidden to take Uſury of any but Strangers.) But latter Ages have produc'd vaſt Alterations in all theſe Reſpects, which having happen'd by inſenſible degrees, may be one reaſon why neither our Legiſlature, nor Courts of Judicature have yet taken ſuch notice thereof, as time and leiſure, with the Tender of proper and practical Methods of Computation, may hereafter induce them to do.

F I N I S.

LONDON: Printed for W. INNYS at the *Prince's Arms* in St. *Paul's Church-Yard*. MDCCXV.